

Q If $P(x) = ax^2 + bx + c$, $Q(x) = -ax^2 + bx + c$
 $a \neq 0$ then $P(x) \cdot Q(x) = 0$ has at least
 how many Real Roots.

Q If α, β are roots of $x^2 + px - r = 0$, $\frac{\alpha}{3}, \beta$
 are roots of $x^2 + qx - r = 0$ then $r = ?$

$$\frac{3(3p-q)(p-3q)}{64}$$

Q Let λ_1, λ_2 be 2 values of λ for which
 Exp. $x^2 + (2-\lambda)x + \lambda - \frac{3}{4}$ becomes a perfect
 sqⁿ then $\lambda_1^2 + \lambda_2^2 = ?$

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Q Value of $x^4 - 8x^3 + 18x^2 - 8x + 2$ if $x = 6 + \frac{\pi}{12}$

1

Q If e^x, e^{-x} are roots of $3x^2 - (a+b)x + 2a = 0$
 $a, b, \lambda \in \mathbb{R}$, $\lambda \neq 0$ then least Integral
 value of $b = ?$

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Q Let a & b are roots of $x^2 + ax + b = 0$
then least value of $x^2 + ax + b = 0$?

$$x^2 + ax + b = 0 \rightarrow \begin{matrix} a \\ b \end{matrix}$$

$$a + b = -a \quad \& \quad a \cdot b = b$$

$$b = -2 \quad a = 1$$

$$2a + b = -2 \quad \therefore \text{Q Eqn} \rightarrow x^2 + x - 2 = 0$$

$$\begin{array}{r} 2a + b = -2 \\ 4a + 2b = -3 \\ \hline -2a = 1 \end{array}$$

$$-2a = 1$$

$$a = -\frac{1}{2}, b = -\frac{1}{2}$$

$$\text{least value} = -\frac{D}{4a}$$

$$= \frac{4 \times 1 \times -2 - (1)^2}{4 \times 1}$$

$$= -\frac{9}{4}$$

Q₂ Let $f(x)$ be a ^{Quad} Poly. S.T.

$$f^2(0) + f^2(1) + f^2(2) + 45 = 10f(0) + 8f(1) + 4f(2)$$

then Range of $f(x)$?

$$f(x) = ax^2 + bx + c = \left[-\frac{x^2}{2} - \frac{x}{2} + 5 \right]$$

$$f(x) = ax^2 + bx + c \quad \text{Range} \left[-\infty, \frac{-D}{4a} \right]$$

$$\left\{ f^2(0) - \boxed{10}f(0) + \boxed{25} \right\} + \left\{ f^2(1) - \boxed{8}f(1) + \boxed{16} \right\} + \left\{ f^2(2) - \boxed{4}f(2) + \boxed{4} \right\} = 0$$

$$(f(0) - 5)^2 + (f(1) - 4)^2 + (f(2) - 2)^2 = 0$$

$$f(0) - 5 = 0 \quad \& \quad f(1) - 4 = 0 \quad \& \quad f(2) - 2 = 0$$

$$f(0) = 5 \Rightarrow c = 5$$

$$f(1) = 4 \Rightarrow a + b + 5 = 4 \Rightarrow a + b = -1$$

$$f(2) = 2 \Rightarrow 4a + 2b + 5 = 2 \Rightarrow 4a + 2b = -3$$

Q find Max/Min value of.

$$f(x) = \frac{4}{(4x^2 + 4x + 9)}$$

$$4x^2 + 4x + 9 \in \left[\frac{(16 - 4 \times 4 \times 9)}{4 \times 4}, \infty \right)$$

(a+)

$$\in \left[\frac{(1-9)}{1}, \infty \right)$$

$$\in [8, \infty)$$

$$+8 \leq 4x^2 + 4x + 9 < \infty$$

$$+ \frac{4}{8} \geq \frac{4}{4x^2 + 4x + 9} > 0$$

$$\rightarrow (0, \frac{1}{2}]$$

Q find Max/Min value of $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$$y = \frac{(3x^2 + 9x + 7) + 10}{(3x^2 + 9x + 7)}$$

$$= 1 + \frac{10}{(3x^2 + 9x + 7)}$$

$$3x^2 + 9x + 7 \in \left[\frac{4 \times 3 \times 7 - 81}{4 \times 3}, \infty \right)$$

$$3x^2 + 9x + 7 \in \left[\frac{7}{4}, \infty \right) \quad \left| \quad 4 \geq \frac{10}{3x^2 + 9x + 7} + 1 > 1 \right.$$

$$\frac{7}{4} \leq 3x^2 + 9x + 7 < \infty$$

$$40 \geq \frac{10}{3x^2 + 9x + 7} > 0$$

$$(1, 41]$$

Range of $\frac{Q}{q}$

$$\frac{1}{7} \leq y \leq 7$$

$$y \in \left[\frac{1}{7}, 7\right]$$

Q. Range of $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$$x^2 y + 3xy + 4y = x^2 - 3x + 4$$

$$x^2(y-1) + x(3y+3) + 4(y+1) = 0 \rightarrow \text{Q in } x$$

Real hogi

$$\begin{aligned} a &\neq 0 \\ y-1 &\neq 0 \\ y &\neq 1 \end{aligned}$$

$$1 = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

$$\begin{aligned} x^2 + 3x + y &= x^2 - 3x + 4 \\ 6x &= 0 \Rightarrow x=0 \end{aligned}$$

$$D \geq 0$$

$$(3y+3)^2 - 4(y-1) \times 4(y+1) \geq 0$$

$$9y^2 + 18y + 9 - 16(y^2 - 2y + 1) \geq 0$$

$$-7y^2 + 50y - 7 \geq 0$$

$$7y^2 - 50y + 7 \leq 0 \Rightarrow 7y^2 - 49y - y + 7 \leq 0$$

$$(7y-1)(y-7) \leq 0$$

Q4: $y = \frac{8m^2x + 46mx + 5}{26m^2x + 86mx + 8}$ find R_f ?

$$y = \frac{(8m^2x + 46mx + 4) + 1}{2(8m^2x + 46mx + 4)}$$

$$= \frac{1}{2} + \frac{1}{2(8m^2x + 46mx + 4)}$$

$$= \frac{1}{2} + \frac{1}{2(8mx + 2)^2} \mid R_f \in \left[\frac{5}{9}, 1\right]$$

$$-1 \leq 8mx \leq 1$$

$$1 \leq 8mx + 2 \leq 3$$

$$1 \leq (8mx + 2)^2 \leq 9$$

$$2 \leq 2(8mx + 2)^2 \leq 18$$

$$\frac{1}{2} + \frac{1}{2} \geq \frac{1}{2(8mx + 2)^2} \geq \frac{1}{18} + \frac{1}{2}$$

$$Q \text{ If } x = \frac{(x+m)^2 - 4mn}{2(x-n)}; \boxed{x \in \mathbb{R}}; m < n$$

find R_f of y .

$$2xy - 2ny = x^2 + 2mx + m^2 - 4mn$$

$$Q \text{ Exp. } \boxed{1} \rightarrow x^2 + 2x(m-y) + m^2 - 4mn + 2ny = 0$$

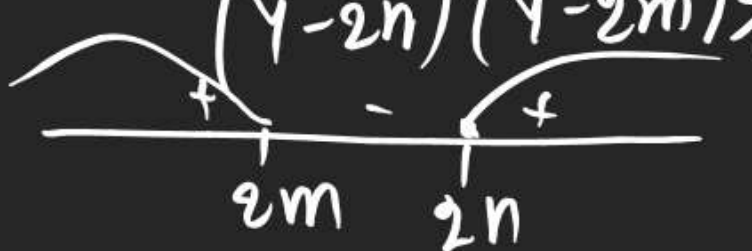
$$D \geq 0$$

$$4(m-y)^2 - 4 \times 1 \times (m^2 - 4mn + 2ny) \geq 0$$

$$y^2 + y^2 - 2my - m^2 + 4mn - 2ny \geq 0$$

$$y^2 - 2my + 4mn - 2ny \geq 0$$

$$y(y-2m) - 2n(y-2m) \geq 0$$

$$(y-2n)(y-2m) \geq 0 \quad y \leq 2m \vee y \geq 2n$$


$$y \in (-\infty, 2m] \cup [2n, \infty)$$

Same find.

$$\frac{Q}{Q}, \frac{Q}{L}, \frac{L}{Q}$$

$$Q \text{ If } y = \frac{x^2 - x + 1}{x^2 - x + 3} \text{ find } R_f?$$

$$x^2y - xy + 3y = x^2 - x + 1$$

$$x^2(y-1) - x(y-1) + (3y-1) = 0$$

$$D \geq 0$$

$$(-(y-1))^2 - 4x(y-1)(3y-1) \geq 0$$

$$y^2 - 2y + 1 - 4(3y^2 - 4y + 1) \geq 0$$

$$-11y^2 + 14y - 3 \geq 0$$

$$11y^2 - 14y + 3 \leq 0$$

$$\boxed{11y^2 - 11y} - \boxed{3y + 3} \leq 0$$

$$(11y-3)(y-1) \leq 0$$

$$3 \leq y \leq 1$$

$$y \in \left[\frac{3}{11}, 1 \right]$$

$$1 = \frac{x^2 - x + 1}{x^2 - x + 3}$$

$$x^2 - x + 3 = x^2 - x + 1$$

$$3 = 1$$

Q If $y = \frac{2x}{1+x^2}$, $x \in \mathbb{R}$

$$x^2 y + y = 2x$$

$$x^2 y - 2x + y = 0$$

$$D \geq 0$$

$$4 - 4x^2 y \geq 0$$

$$y^2 - 1 \leq 0$$

$$-1 \leq y \leq 1$$

$$y \neq 0$$

$$0 = \frac{2x}{1+x^2}$$

$$2x = 0$$

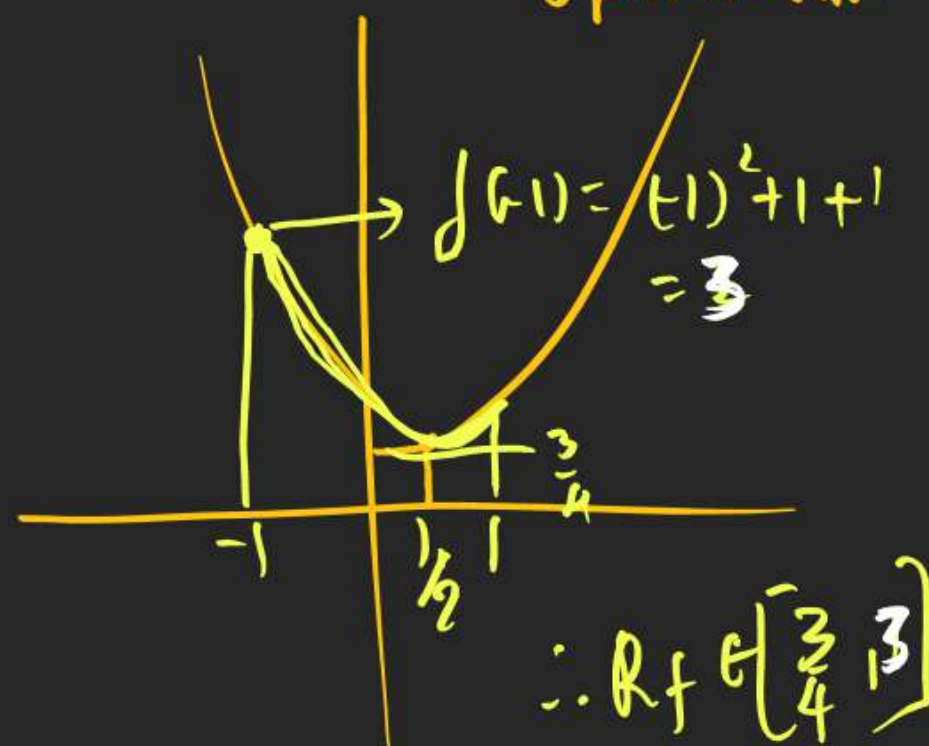
$$x = 0$$

Range of $f(x) = y^2 - y + 1$?

$$f(y) = y^2 - y + 1$$

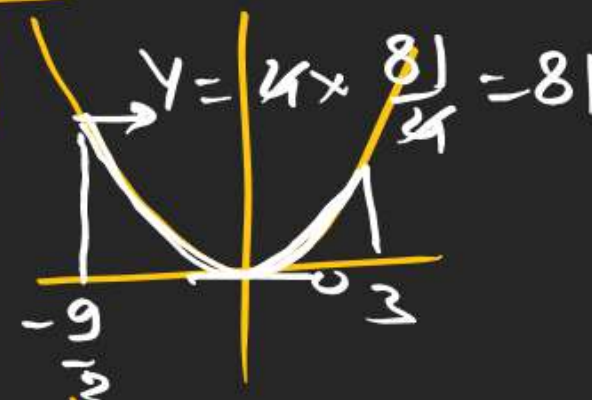
$$V = \left(\frac{1}{2}, \frac{3}{4}\right)$$

Upward Par.



Q If $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} \leq 4$ then least & highest

value of $4x^2$



$$\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \leq 0$$

$$\frac{6x^2 - 5x - 3 - 4x^2 + 8x - 24}{x^2 - 2x + 6} \leq 0$$

$$\frac{2x^2 + 3x - 27}{x^2 - 2x + 6} \leq 0$$

$D = (4 - 24) > 0$

$$2x^2 + 3x - 27 \leq 0$$

$$2x^2 - 6x + 9x - 27 \leq 0$$

$$(2x + 9)(x - 3) \leq 0$$

$$-\frac{9}{2} \leq x \leq 3$$

$$\therefore R_f \in [0, 81]$$

$$\textcircled{1} \quad y = \frac{2x-3}{3x+4} \quad R+? \quad \left(\frac{L}{L} \right)$$

$$3xy + 4y = 2x - 3.$$

$$x(3y-2) = -4y-3.$$

$$x = \frac{4y+3}{2-3y} \quad \text{Domain is } \mathbb{R} \text{ also}$$

$$2-3y \neq 0$$

$$y \neq \frac{2}{3}$$

$$\therefore y \in (-\infty, \infty) - \left\{ \frac{2}{3} \right\}$$

$$y = \frac{ax+b}{cx+d}$$

$$R_f = \mathbb{R} - \left\{ \frac{a_2}{c} \right\}$$

$$\textcircled{1} R + \sigma_f y = \frac{x^2 - x - 2}{x^2 + 3x} \quad x \in \mathbb{R}.$$

$$x^2 y + 3x(y) = x^2 - x - 2$$

$$x^2(y-1) + x(3y+1) + 2 = 0$$

$$D \geq 0$$

$$3y+1 \leq 4(y-1) \times 2 \geq 0$$

$$9y^2 + 6y + 1 - 8y + 8 \geq 0$$

$$\underline{9y^2 - 2y + 9 \geq 0}$$

$$D = 4 - 4 \times 81 = -ve$$

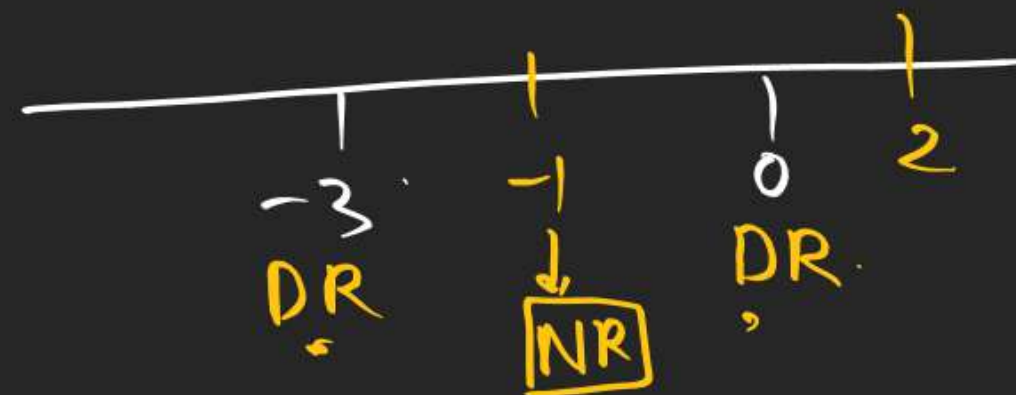
$$9y^2 - 2y + 9 > 0 \text{ for all } y \in \mathbb{R}.$$

$$\underline{y \in \mathbb{R}}$$

M2

$$y = \frac{(x-2)(x+1)}{(x)(x+3)} \rightarrow NR$$

$$(x)(x+3) \rightarrow DR$$



Exactly 1 Root of NR lying betⁿ.

2 Roots of DR

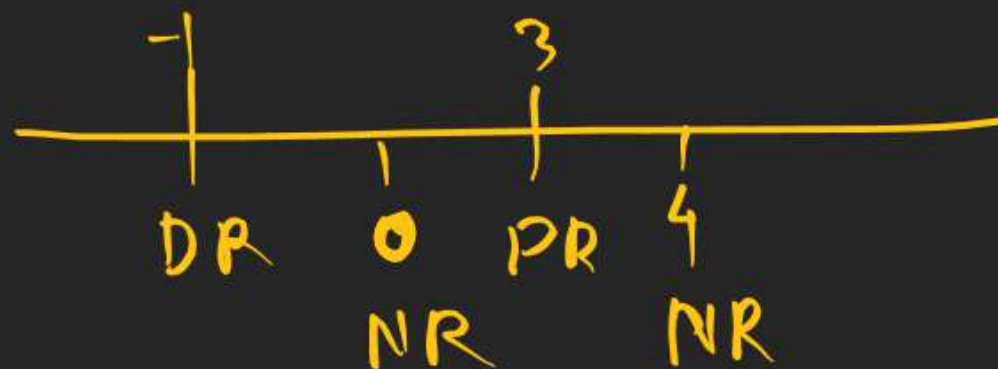
$R \notin \mathbb{R}.$

$$Q \quad y = \frac{(x-1)}{(x)(x-2)} \text{ find } R_f$$



Exactly 1 Root of NR
lying betⁿ 2 Roots of DR.
∴ $y \in R$

$$Q \quad y = \frac{(x)(x-4)}{(x+1)(x-3)} \text{ find } R_f; x \in R?$$



Exactly 1 Root of NR lying betⁿ 2 Roots of DR

Common Roots

① We try to find Common Root

in 2 given Eqⁿ always.

One Common Root

(2) $a_1x^2 + b_1x + c_1 = 0$ α

$a_2x^2 + b_2x + c_2 = 0$ α

here α is a com. Root for both Eqⁿ.

$$a_1\alpha^2 + b_1\alpha + c_1 = 0$$

$$a_2\alpha^2 + b_2\alpha + c_2 = 0$$

$$\frac{\alpha^2}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{-\alpha}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{-\alpha}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\alpha = \frac{b_2c_1 - b_1c_2}{a_1c_2 - a_2c_1}$$

$$\alpha = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

Q If $3x^2 + 4mx + 2 = 0$ & $2x^2 + 3x - 2 = 0$ have

one common root then $m = ?$

Both Root com.

$$2x^2 + 3x - 2 = 0$$

$$2x^2 + 4x - x - 2 = 0$$

$$2x(x+2) - 1(x+2) = 0$$

$$x = 1, x = -2$$

$$D = 9 + 32$$

$$= 41$$

Irr. Roots

They comes in Conjugate Pair

Q Poori de Rkhi ho

$D = -ve$

$\Rightarrow$

$D = \text{Prime}$

No

LFDA

(B) Both Root common. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$\begin{aligned} a_1 x^2 + b_1 x + c_1 &= 0 \quad \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \\ a_2 x^2 + b_2 x + c_2 &= 0 \quad \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \end{aligned}$$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

$$x^2 - 4x + 5 = 0$$

$$3x^2 - 12x + 15 = 0$$

$$\frac{1}{3} = \frac{-4}{-12} = \frac{5}{15}$$

$$3x^2 + 4mx + 2 = 0$$

$$x = \frac{1}{2}$$

$$\frac{3}{4} + \frac{4m}{2} + 2 = 0$$

$$2m = -2 - \frac{3}{4} = -\frac{11}{4}$$

$$m = -\frac{11}{8}$$

$$x = -2$$

$$12 - 8m + 2 = 0$$

$$8m = 14$$

$$m = \frac{7}{4}$$